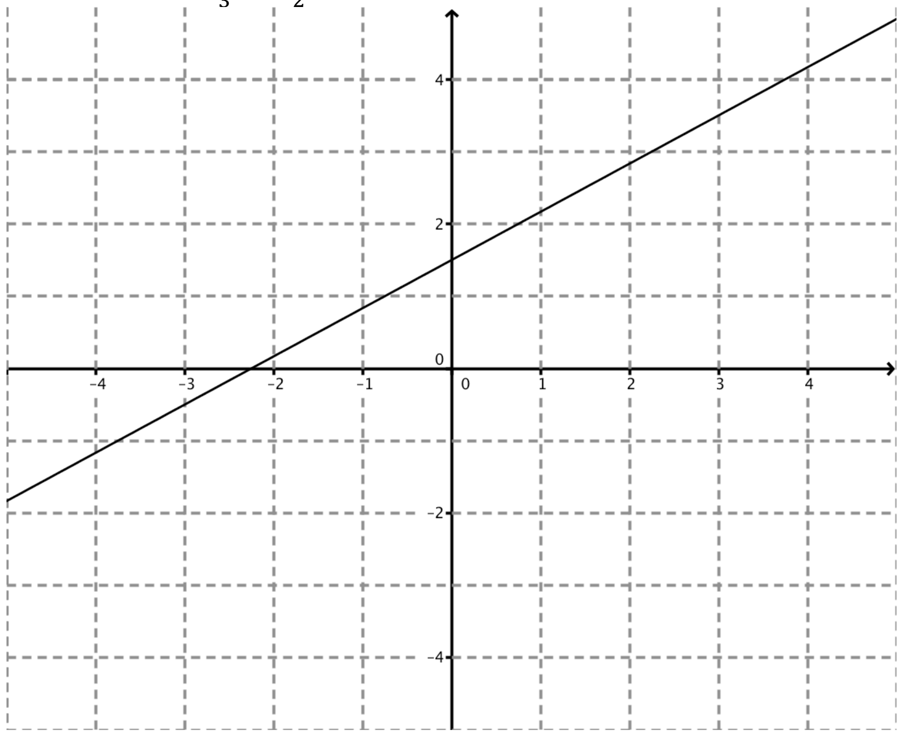
Consider the function $f(x) = \frac{2}{3}x + \frac{3}{2}$. Its graph is provided below.



By looking at the graph, it is pretty easy to see that its *y*-intercept is: (,).

In fact, we can verify that our *y*-intercept is correct by plugging in $x = 0$. Doing, so, we get:

$$f(0) = \_\_\_\_\_\_$$

By looking at the graph, we can get a good estimate of the *x*-intercept is: (,).

Let's find the exact answer by setting $f(x) = 0$ and solving for *x*. Complete the remaining steps below:

$$\frac{2}{3}x + \frac{3}{2} = 0$$

Along with finding intercepts, we can also find when $f(x)$ will output various values. For example, looking at the graph, we see that $f(x) = 3$ a little after $x = 2$. We can solve exactly by solving for *x* in the following equation. Complete the remaining steps below:

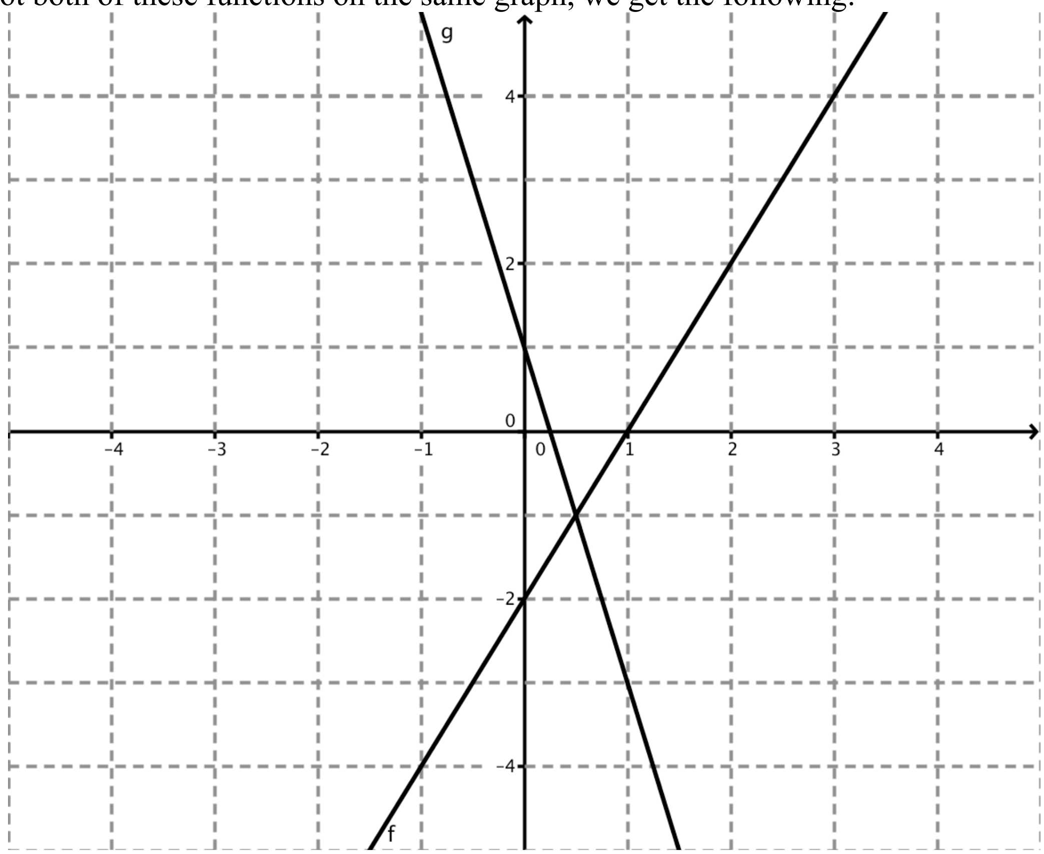
$$\frac{2}{3}x + \frac{3}{2} = 3$$

We can even use graphs of linear functions to solve more complicated linear equations.

Let's consider the linear equation $2x - 2 = 1 - 4x$. We can visualize this by considering the two linear functions

$$f(x) = 2x - 2$$
$$g(x) = 1 - 4x$$

If we plot both of these functions on the same graph, we get the following:



These two lines seem to intersect around $\left(\frac{1}{2}, -1\right)$. The x-coordinate of this point corresponds to when f(x) = g(x); in other words, it's the x-value for which $2x - 2 = 1 - 4x$, which is exactly the linear equation we want to solve!

Now, use algebra to solve for x:

$$2x - 2 = 1 - 4x$$

Did you get the same answer as we expected from looking at the graph?

Before we move on to solving more complicated equations graphically and algebraically, let's be sure that we've mastered solving *linear equations* algebraically.

In the following, solve for *x*. Be sure to show all your work.

1. Solve for x if $2x - 7 = 3x + 2$.

2. Solve for x if $\frac{1}{2}x + 5 = 9$.

3. Solve for x if $3x - \frac{7}{2} = \frac{11}{2}$.

4. Solve for x if $\frac{2}{3}x - \frac{5}{2} = \frac{9}{2}$.

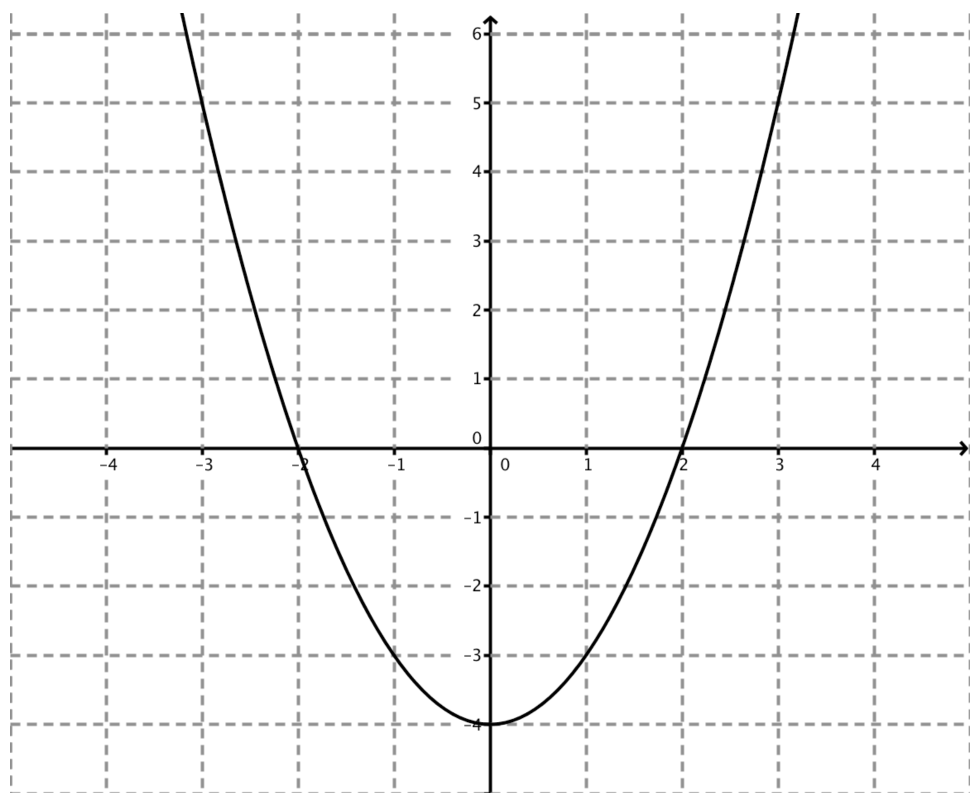
We just saw that there are actually two ways to solve linear equations: algebraically and graphically. However, linear equations are some of the easier ones in mathematics. So, with a little patience, we can essentially solve them all with a little algebra.

More complicated equations are harder to solve algebraically, so we should use our graphical approach.

For example, let's consider what is called a *quadratic equation*. Here's one:

$$x^2 - 4 = 0.$$

First off, this is called a *quadratic equation* because of the x^2 term. Next quarter, we'll learn how to solve such equations algebraically. In the mean time, let's solve these graphically. Here is a plot of the function $f(x) = x^2 - 4$.



Since $f(x) = x^2 - 4$, we can solve $x^2 - 4 = 0$ by seeing when $f(x) = 0$ on our graph. Doing so yields two answers:

x = ____ and ____.

Now that we have the graph, we can solve other equations using $x^2 - 4$. For example, we can see when $x^2 - 4 = -3$ by seeing when $f(x) = -3$. Doing so yields two answers:

x = ____ and ____.

Use the graph of $f(x) = x^2 - 4$ to solve the below equations:

$x^2 - 4 = 5$

x = ____ and ____

$x^2 - 4 = -4$

x = ______

Many times in quadratic equations, the solutions we get for x are not numbers that are easy to write down. But we can at least approximate them.

Use the graph of $f(x) = x^2 - 4$ to approximate the solutions to:

$x^2 - 4 = 1$

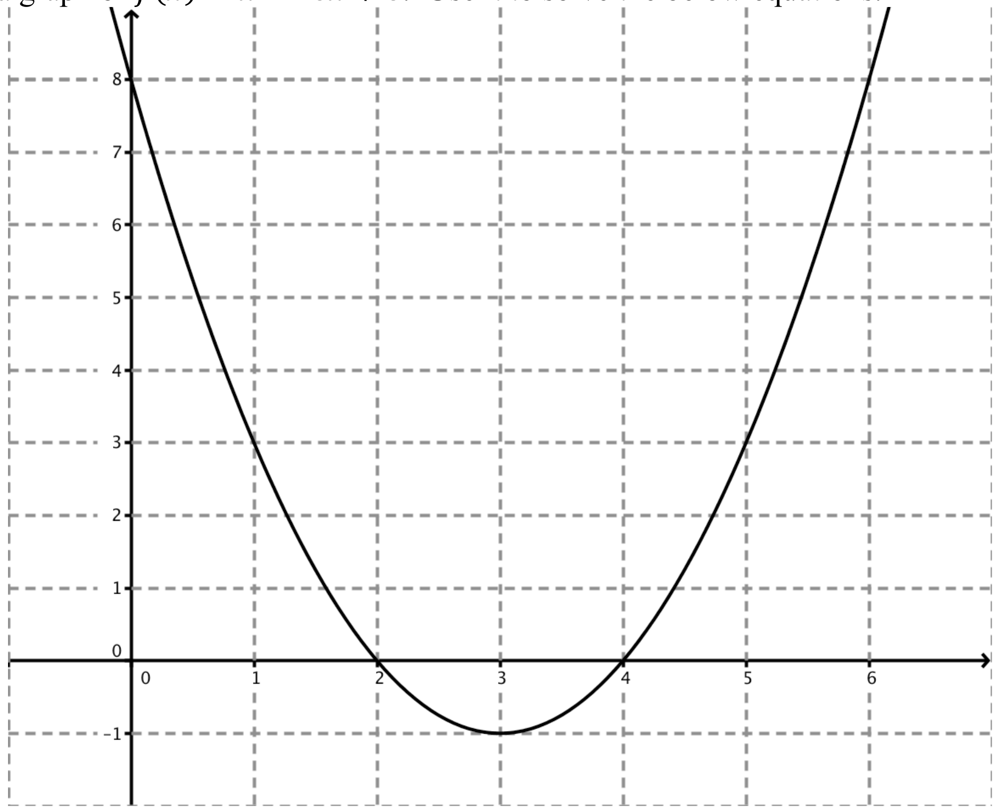
x = _____ and _____

$x^2 - 4 = 3$

x = _____ and _____

Sometimes, we can use graphs to tell us when there is no solution to an equation. Below, explain why there is no solution to $x^2 - 4 = -5$.

Below is a graph of $f(x) = x^2 - 6x + 8$. Use it to solve the below equations.



1. $x^2 - 6x + 8 = 0$

2. $x^2 - 6x + 8 = -1$

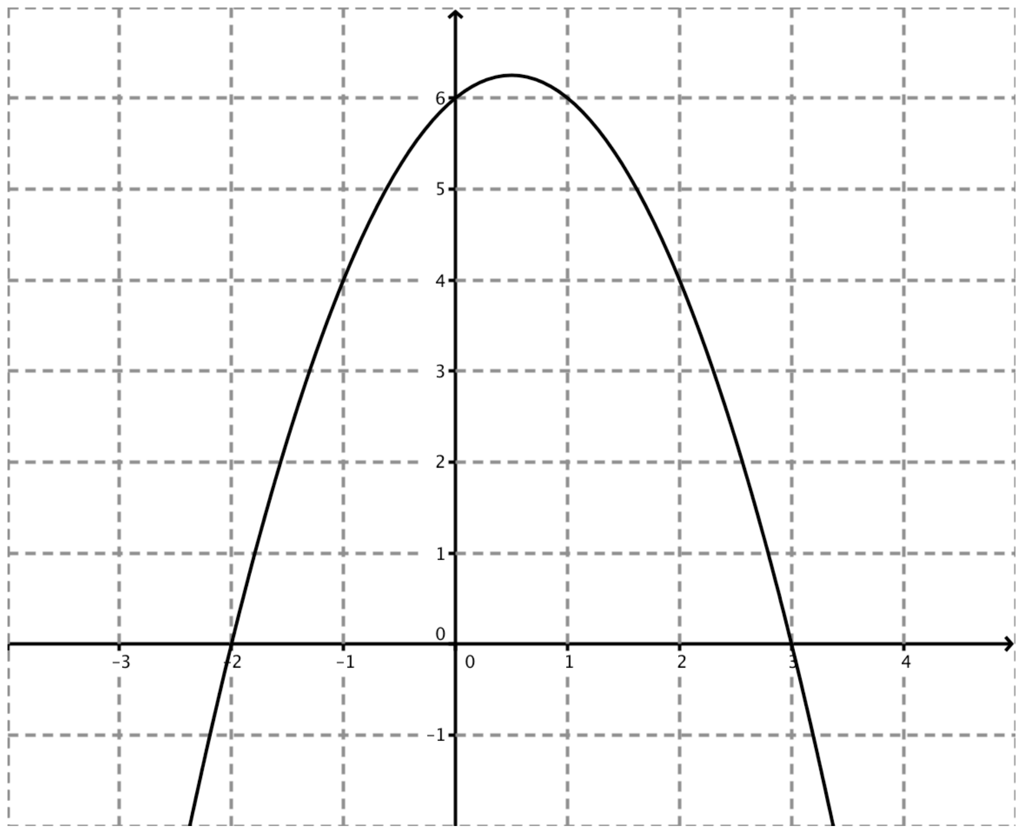
3. $x^2 - 6x + 8 = 8$

4. $x^2 - 6x + 8 = 3$

5. Approximate the solutions to $x^2 - 6x + 8 = 6$

6. Explain why $x^2 - 6x + 8 = -2$ has no solutions.

Below is a graph of $f(x) = -x^2 + x + 8$. Use it to solve the below equations.



7. $-x^2 + x + 8 = 0$

8. $-x^2 + x + 8 = 6$

9. $-x^2 + x + 8 = 4$

10. Approximate the solutions to $-x^2 + x + 8 = -1$.

11. Explain why $-x^2 + x + 8 = 10$ has no solutions.

At each of the stations, use the graph to solve one the given equations. If an equation has no solution, write "No solution" and explain why no solution exists.

Station 1

$x^2 - 36 = 0$ $x =$ ________________

$x^2 - 36 = -20$ $x =$ ________________

$x^2 - 36 = -20$ $x =$ ________________

Station 2

$x^2 + x - 6 = 0$ $x =$ ________________

$x^2 + x - 6 = -4$ $x =$ ________________

$x^2 + x - 6 = 14$ $x =$ ________________

Station 3

$2x - 14 = -6$ $x =$ ________________

$2x - 14 = 0$ $x =$ ________________

$2x - 14 = -16$ $x =$ ________________

Station 4

$x^2 - 9 = 16$ $x =$ ________________

$x^2 - 9 = -11$ $x =$ ________________

$x^2 - 9 = -8$ $x =$ ________________

Station 5

$x^2 + 4x - 5 = 0$ $x =$ ________________

$x^2 + 4x - 5 = -9$ $x =$ ________________

$x^2 + 4x - 5 = -10$ $x =$ ________________

Station 6

$x^3 + 2x^2 + x - 4 = 0$ $x =$ ________________

$x^3 + 2x^2 + x - 4 = -4$ $x =$ ________________

$x^3 + 2x^2 + x - 4 = -6$ $x =$ ________________

Station 7

$x^2 - 2x - 15 = 0$ $x =$ ________________

$x^2 - 2x - 15 = -20$ $x =$ ________________

$x^2 - 2x - 15 = -12$ $x =$ ________________

Station 8

$x^2 - 25 = 0$ $x =$ ________________

$x^2 - 25 = -16$ $x =$ ________________

$x^2 - 25 = -25$ $x =$ ________________

Station 9

$x^2 - x - 2 = 0$ $x =$ ________________

$x^2 - x - 2 = 4$ $x =$ ________________

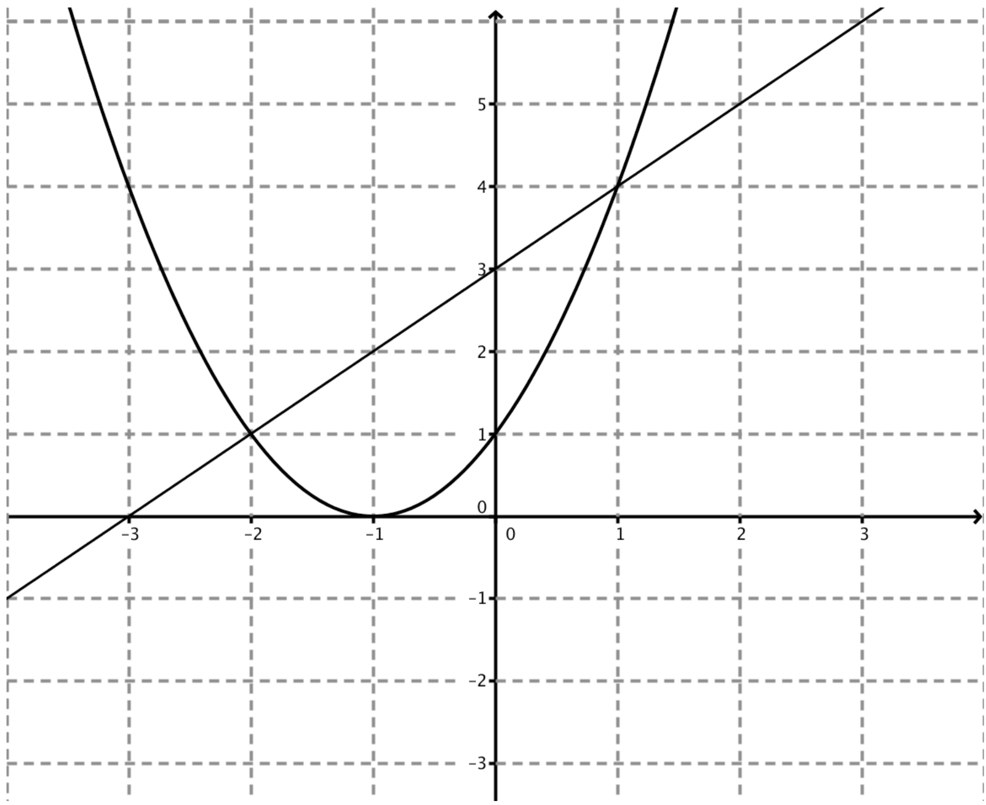
$x^2 - x - 2 = 10$ $x =$ ________________

We can also solve even more complicated equations by using two graphs of functions.

For example, if we wanted to solve the complicated equation

$$x^2 + 2x + 1 = x + 3$$

Below, we have the graphs of $f(x) = x^2 + 2x + 1$ and $g(x) = x + 3$



Thus, we can solve $x^2 + 2x + 1 = x + 3$ by finding which x-coordinates satisfy $f(x) = g(x)$.

Using the graph, we can solve $x^2 + 2x + 1 = x + 3$ to get: $x =$ _______________